OCR

Oxford Cambridge and RSA

Monday 23 May 2022 – Afternoon

AS Level Further Mathematics B (MEI)

Y411/01 Mechanics a

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

OCR is an exempt Charity

Turn over

Answer **all** the questions.

1 (a) **Fig. 1.1** and **Fig. 1.2** show rigid rods with forces acting as marked. The diagrams are to scale, and in each figure the side length of a grid square is 1 metre.
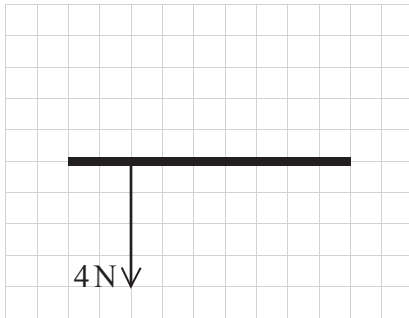

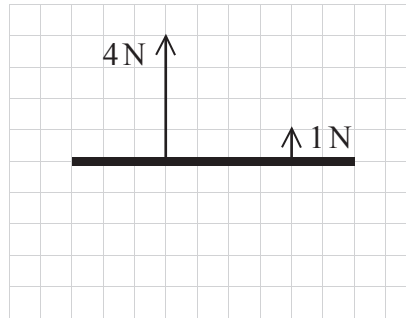**Fig. 1.1**   **Fig. 1.2**

- On the copy of **Fig. 1.1** in the Printed Answer Booklet, add, to scale, a force so that the overall system represents an anti-clockwise couple of magnitude 24 N m.
- On the copy of **Fig. 1.2** in the Printed Answer Booklet, add, to scale, a force so that the overall system represents a clockwise couple of magnitude 1 N m.

[3]

(b) **Fig. 1.3** shows a rectangular lamina with two coplanar forces acting as marked. Each grid square has side length 1 m.
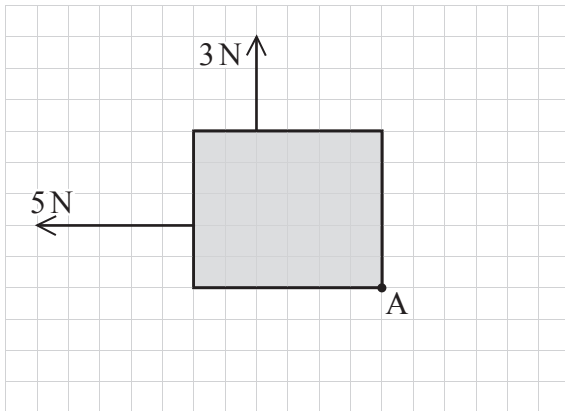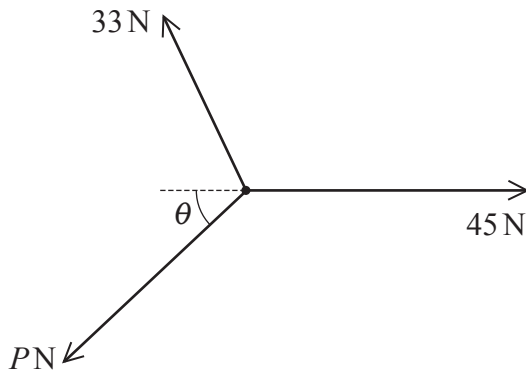

Fig. 1.3

A third coplanar force, of magnitude T N, acts at A so that the resultant force on the lamina is zero.

(i) Calculate the value of T. **[1]**

(ii) Determine the magnitude and direction of the couple represented by this system of three forces. **[3]**

2 Three forces, of magnitudes 33 N, 45 N and P N, act at a point in the directions shown in the diagram. The system is in equilibrium.

(a) Draw a triangle of forces for the system shown above. Your diagram should include the magnitudes of the forces (33 N, 45 N and P N) and angle θ. **[2]**

(b) If $P = 38$, find, in degrees, the value of θ. **[2]**

(c) If $\theta = 40°$, determine the possible values for P. **[3]**

3   **Fig. 3.1** shows a thin rectangular frame ABCD, with part of it filled by a triangular lamina ABD. AD = 30 cm and AB = x cm. Together they form the composite structure S.

The centre of mass of S lies at a point M, 16.5 cm from AD and 11.7 cm from AB.
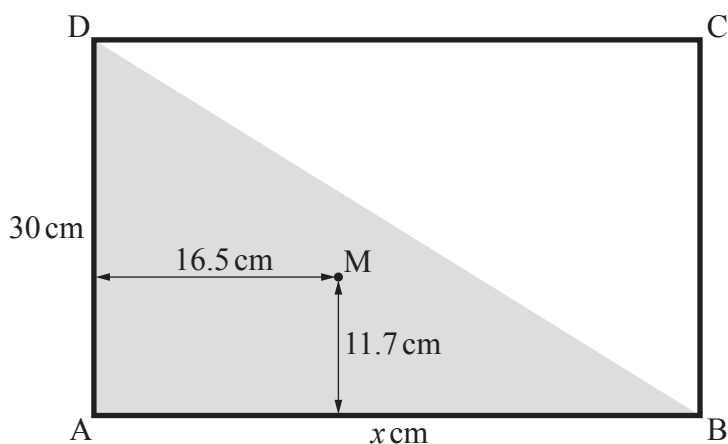

Fig. 3.1

The frame and the triangular lamina are both uniform but made of different materials. The mass of the frame is 1.7 kg.

(a)   Show that the triangular lamina has a mass of 3.3 kg.   **[3]**

(b)   Determine the value of x, correct to **3** significant figures.   **[2]**

One end of a light inextensible string is attached to S at D. The other end is attached to a fixed point on a vertical wall. For S to hang in equilibrium with AD vertical, a force of magnitude Q N is applied to S as shown in **Fig. 3.2**. The line of action of this force lies in the same plane as S. The string is taut and lies in the same plane as S at an angle ϕ to the downward vertical.
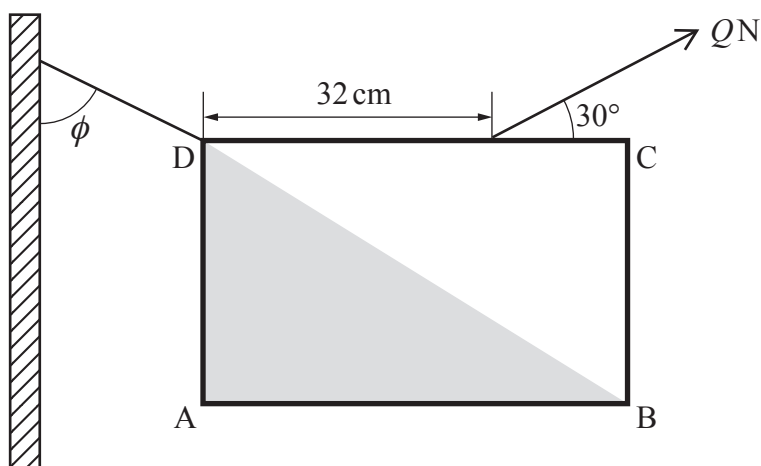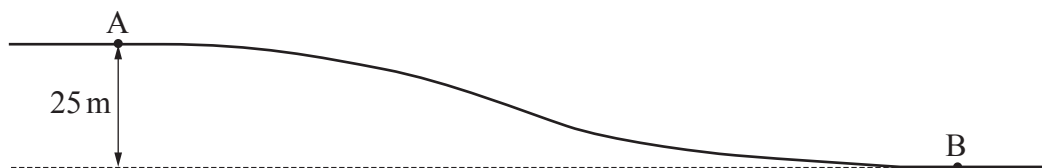

Fig. 3.2

(c)   By taking moments about D, show that $Q = 50.5$, correct to **3** significant figures.   **[2]**

(d)   Determine, in degrees, the value of ϕ.   **[3]**

4 The diagram shows two points A and B on a snowy slope. A is a vertical distance of 25 m above B.

A rider and snowmobile, with a combined mass of 240 kg, start at the top of the slope, heading in the direction of B. As the snowmobile passes A, with a speed of $3\,\mathrm{m\,s^{-1}}$, the rider switches off the engine so that the snowmobile coasts freely. When the snowmobile passes B, it has a speed of $18\,\mathrm{m\,s^{-1}}$.

The resistances to motion can be modelled as a single, constant force of magnitude 120 N.

(a) Calculate the distance the snowmobile travels from A to B. **[3]**

The rider now turns the snowmobile around and brings it back to B, so that it faces up the slope. Starting from rest, the snowmobile ascends the slope so that it passes A with a speed of $7\,\mathrm{m\,s^{-1}}$. It takes 30 seconds for the snowmobile to travel from B to A. The resistances to motion can still be modelled as a single, constant force of magnitude 120 N.

(b) Show that the snowmobile develops an average power of 2856 W during this time. **[2]**

The snowmobile can develop a maximum power of 6000 W. At a later point in the journey, the rider and snowmobile reach a different slope inclined at 12° to the horizontal. The resistances to motion can still be modelled as a single, constant force of magnitude 120 N.

(c) Determine the maximum speed with which the rider and snowmobile can ascend. **[3]**

The power developed by a vehicle is sometimes given in the non-SI unit *mechanical horsepower* (hp). 1 hp is the power required to lift 550 pounds against gravity, starting and ending at rest, by 1 foot in 1 second.

(d) Given that 1 metre ≈ 3.28 feet and 1 kg ≈ 2.2 pounds, determine the number of watts that are equivalent to 1 hp. **[2]**

5 **Fig. 5.1** shows a small smooth sphere A at rest on a smooth horizontal surface. At both ends of the surface is a smooth vertical wall.

Fig. 5.1

Sphere A is projected directly towards the left-hand wall at a speed of $5\,\mathrm{m\,s}^{-1}$. Sphere A collides directly with the left-hand wall, rebounds, then collides directly with the right-hand wall. After this second collision A has a speed of $3.2\,\mathrm{m\,s}^{-1}$.

(a) Explain how it can be deduced that the collision between A and the left-hand wall was **not** inelastic. **[1]**

The coefficient of restitution between A and each wall is e.

(b) Calculate the value of e. **[2]**

Sphere A is now brought to rest and a second identical sphere B is placed on the surface. The surface is 1 m long, and A and B are positioned so that they are both 0.5 m from each wall, as shown in **Fig. 5.2**.
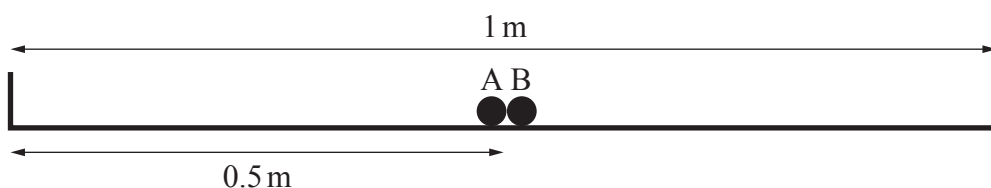

Fig. 5.2

Sphere A is projected directly towards the left-hand wall at a speed of $0.2\,\mathrm{m\,s}^{-1}$. At the same time, B is projected directly towards the right-hand wall at a speed of $0.3\,\mathrm{m\,s}^{-1}$. You may assume that the duration of impact of a sphere and a wall is negligible.

(c) Calculate the distance of A and B from the left-hand wall when they meet again. **[3]**

6 A block B of mass m kg rests on a rough slope inclined at angle α to the horizontal. The coefficient of friction between B and the slope is $\frac{5}{9}$.

(a) When B is in limiting equilibrium, show that $\tan\alpha = \frac{5}{9}$. **[3]**

(b) If $\alpha = 40°$, determine the acceleration of B down the slope. **[2]**

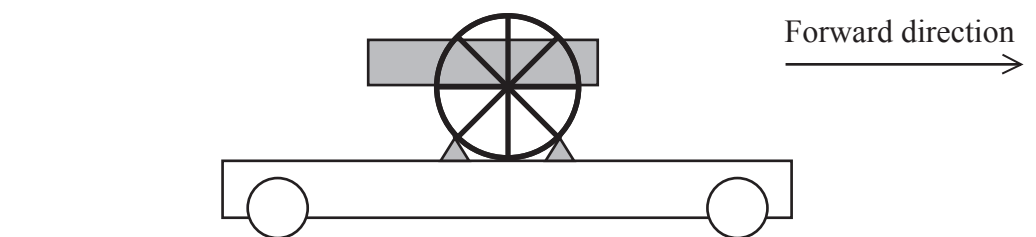A horizontal force of magnitude P N is now applied to B, as shown in the diagram below. At first B is at rest.
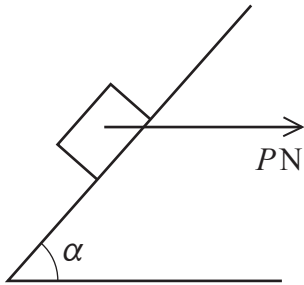
P is gradually increased.

(c) Show that, for B to slide on the slope,

$$P\left(\cos\alpha - \tfrac{5}{9}\sin\alpha\right) > mg\left(\tfrac{5}{9}\cos\alpha + \sin\alpha\right).$$ **[3]**

(d) Determine, in degrees, the least value of α for which B will not slide no matter how large P becomes. **[2]**

7 The diagram shows a cannon fixed to a trolley. The trolley runs on a smooth horizontal track.

Forward direction

A driver boards the trolley with two cannon balls. The combined mass of the trolley, driver, cannon and cannon balls is 320 kg. Each cannon ball has a mass of 5 kg. Initially the trolley is at rest.

A force of 480 N acts on the trolley in the forward direction for 4 seconds.

(a) (i) Calculate the magnitude of the impulse of the force on the trolley. **[1]**

    (ii) Calculate the speed of the trolley after the force stops acting. **[1]**

The driver now fires a cannon ball horizontally in the backward direction. The cannon ball and cannon separate at a rate of $90 \, \text{m s}^{-1}$.

(b) Show that, after the firing of the cannon ball, the trolley moves with a speed of $7.41 \, \text{m s}^{-1}$, correct to **3** significant figures. **[3]**

The driver now reverses the direction of the cannon and fires the second cannon ball horizontally in the forward direction. Again, the cannon ball and cannon separate at a rate of $90 \, \text{m s}^{-1}$.

(c) Calculate the overall percentage change in the kinetic energy of the trolley (alone) from before the first cannon ball is fired to after the second is fired, giving your answer correct to **2** decimal places. You should make clear whether the change in kinetic energy is a gain or a loss. **[4]**

(d) Give a reason why one of the modelling assumptions that was required in answering parts **(a)**, **(b)** and **(c)** may not have been appropriate. **[1]**

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END OF QUESTION PAPER

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